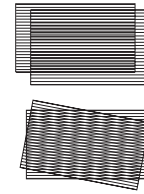


2.6 Mensuration

Moiré fringes^a

Parallel pattern	$d_M = \left \frac{1}{d_1} - \frac{1}{d_2} \right ^{-1}$ (2.244)	d_M Moiré fringe spacing $d_{1,2}$ grating spacings
Rotational pattern ^b	$d_M = \frac{d}{2 \sin(\theta/2) }$ (2.245)	d common grating spacing θ relative rotation angle ($ \theta \leq \pi/2$)

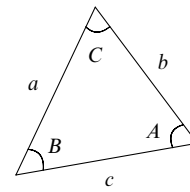


^aFrom overlapping linear gratings.

^bFrom identical gratings, spacing d , with a relative rotation θ .

Plane triangles

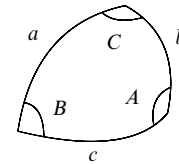
Sine formula ^a	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (2.246)	
Cosine formulas	$a^2 = b^2 + c^2 - 2bc \cos A$ (2.247)	
	$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ (2.248)	
	$a = b \cos C + c \cos B$ (2.249)	
Tangent formula	$\tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2}$ (2.250)	
Area	area $= \frac{1}{2} ab \sin C$ (2.251)	
	$= \frac{a^2 \sin B \sin C}{2 \sin A}$ (2.252)	
	$= [s(s-a)(s-b)(s-c)]^{1/2}$ (2.253)	
	where $s = \frac{1}{2}(a+b+c)$ (2.254)	



^aThe diameter of the circumscribed circle equals $a/\sin A$.

Spherical triangles^a

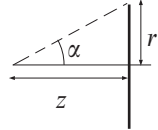
Sine formula	$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$ (2.255)	
Cosine formulas	$\cos a = \cos b \cos c + \sin b \sin c \cos A$ (2.256)	
	$\cos A = -\cos B \cos C + \sin B \sin C \cos a$ (2.257)	
Analogue formula	$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A$ (2.258)	
Four-parts formula	$\cos a \cos C = \sin a \cot b - \sin C \cot B$ (2.259)	
Area ^b	$E = A + B + C - \pi$ (2.260)	



^aOn a unit sphere.

^bAlso called the "spherical excess."

Perimeter, area, and volume

Perimeter of circle	$P = 2\pi r$	(2.261)	P perimeter r radius
Area of circle	$A = \pi r^2$	(2.262)	A area
Surface area of sphere ^a	$A = 4\pi R^2$	(2.263)	R sphere radius
Volume of sphere	$V = \frac{4}{3}\pi R^3$	(2.264)	V volume
Perimeter of ellipse ^b	$P = 4aE(\pi/2, e)$	(2.265)	a semi-major axis b semi-minor axis E elliptic integral of the second kind (p. 45)
	$\simeq 2\pi \left(\frac{a^2 + b^2}{2} \right)^{1/2}$	(2.266)	e eccentricity ($= 1 - b^2/a^2$)
Area of ellipse	$A = \pi ab$	(2.267)	
Volume of ellipsoid ^c	$V = 4\pi \frac{abc}{3}$	(2.268)	c third semi-axis
Surface area of cylinder	$A = 2\pi r(h + r)$	(2.269)	h height
Volume of cylinder	$V = \pi r^2 h$	(2.270)	
Area of circular cone ^d	$A = \pi r l$	(2.271)	l slant height
Volume of cone or pyramid	$V = A_b h/3$	(2.272)	A_b base area
Surface area of torus	$A = \pi^2 (r_1 + r_2)(r_2 - r_1)$	(2.273)	r_1 inner radius r_2 outer radius
Volume of torus	$V = \frac{\pi^2}{4} (r_2^2 - r_1^2)(r_2 - r_1)$	(2.274)	
Area ^d of spherical cap, depth d	$A = 2\pi R d$	(2.275)	d cap depth
Volume of spherical cap, depth d	$V = \pi d^2 \left(R - \frac{d}{3} \right)$	(2.276)	Ω solid angle z distance from centre α half-angle subtended
Solid angle of a circle from a point on its axis, z from centre	$\Omega = 2\pi \left[1 - \frac{z}{(z^2 + r^2)^{1/2}} \right]$	(2.277)	
	$= 2\pi(1 - \cos \alpha)$	(2.278)	

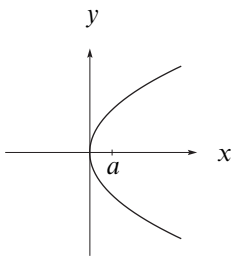
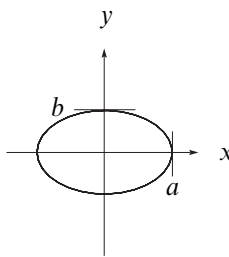
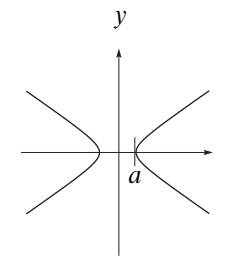
^aSphere defined by $x^2 + y^2 + z^2 = R^2$.

^bThe approximation is exact when $e=0$ and $e \simeq 0.91$, giving a maximum error of 11% at $e=1$.

^cEllipsoid defined by $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$.

^dCurved surface only.

Conic sections

			
	<i>parabola</i>	<i>ellipse</i>	<i>hyperbola</i>
equation	$y^2 = 4ax$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
parametric form	$x = t^2/(4a)$ $y = t$	$x = acost$ $y = b \sin t$	$x = \pm a \cosh t$ $y = b \sinh t$
foci	$(a, 0)$	$(\pm \sqrt{a^2 - b^2}, 0)$	$(\pm \sqrt{a^2 + b^2}, 0)$
eccentricity	$e = 1$	$e = \frac{\sqrt{a^2 - b^2}}{a}$	$e = \frac{\sqrt{a^2 + b^2}}{a}$
directrices	$x = -a$	$x = \pm \frac{a}{e}$	$x = \pm \frac{a}{e}$

Platonic solids^a

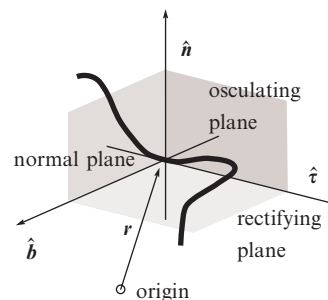
<i>solid</i> (<i>faces, edges, vertices</i>)	<i>volume</i>	<i>surface area</i>	<i>circumradius</i>	<i>inradius</i>
tetrahedron (4, 6, 4)	$\frac{a^3 \sqrt{2}}{12}$	$a^2 \sqrt{3}$	$\frac{a \sqrt{6}}{4}$	$\frac{a \sqrt{6}}{12}$
cube (6, 12, 8)	a^3	$6a^2$	$\frac{a \sqrt{3}}{2}$	$\frac{a}{2}$
octahedron (8, 12, 6)	$\frac{a^3 \sqrt{2}}{3}$	$2a^2 \sqrt{3}$	$\frac{a}{\sqrt{2}}$	$\frac{a}{\sqrt{6}}$
dodecahedron (12, 30, 20)	$\frac{a^3 (15 + 7\sqrt{5})}{4}$	$3a^2 \sqrt{5(5 + 2\sqrt{5})}$	$\frac{a}{4} \sqrt{3}(1 + \sqrt{5})$	$\frac{a}{4} \sqrt{\frac{50 + 22\sqrt{5}}{5}}$
icosahedron (20, 30, 12)	$\frac{5a^3 (3 + \sqrt{5})}{12}$	$5a^2 \sqrt{3}$	$\frac{a}{4} \sqrt{2(5 + \sqrt{5})}$	$\frac{a}{4} \left(\sqrt{3} + \sqrt{\frac{5}{3}} \right)$

^aOf side a . Both regular and irregular polyhedra follow the Euler relation, faces – edges + vertices = 2.

Curve measure

Length of plane curve	$l = \int_a^b \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{1/2} dx \quad (2.279)$	a start point b end point $y(x)$ plane curve l length
Surface of revolution	$A = 2\pi \int_a^b y \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{1/2} dx \quad (2.280)$	A surface area
Volume of revolution	$V = \pi \int_a^b y^2 dx \quad (2.281)$	V volume
Radius of curvature	$\rho = \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2} \left(\frac{d^2y}{dx^2} \right)^{-1} \quad (2.282)$	ρ radius of curvature

Differential geometry^a

Unit tangent	$\hat{\tau} = \frac{\dot{\mathbf{r}}}{ \dot{\mathbf{r}} } = \frac{\dot{\mathbf{r}}}{v} \quad (2.283)$	τ tangent $\mathbf{r}$ curve parameterised by $\mathbf{r}(t)$ v $ \dot{\mathbf{r}}(t) $
Unit principal normal	$\hat{\mathbf{n}} = \frac{\ddot{\mathbf{r}} - \dot{v}\hat{\tau}}{ \ddot{\mathbf{r}} - \dot{v}\hat{\tau} } \quad (2.284)$	$\mathbf{n}$ principal normal
Unit binormal	$\hat{\mathbf{b}} = \hat{\tau} \times \hat{\mathbf{n}} \quad (2.285)$	$\mathbf{b}$ binormal
Curvature	$\kappa = \frac{ \dot{\mathbf{r}} \times \ddot{\mathbf{r}} }{ \dot{\mathbf{r}} ^3} \quad (2.286)$	κ curvature
Radius of curvature	$\rho = \frac{1}{\kappa} \quad (2.287)$	ρ radius of curvature
Torsion	$\lambda = \frac{\dot{\mathbf{r}} \cdot (\ddot{\mathbf{r}} \times \dddot{\mathbf{r}})}{ \dot{\mathbf{r}} \times \ddot{\mathbf{r}} ^2} \quad (2.288)$	λ torsion
Frenet's formulas	$\dot{\hat{\tau}} = \kappa v \hat{\mathbf{n}} \quad (2.289)$ $\dot{\hat{\mathbf{n}}} = -\kappa v \hat{\tau} + \lambda v \hat{\mathbf{b}} \quad (2.290)$ $\dot{\hat{\mathbf{b}}} = -\lambda v \hat{\mathbf{n}} \quad (2.291)$	

^aFor a continuous curve in three dimensions, traced by the position vector $\mathbf{r}(t)$.